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Influence of boundary conditions of digressively proportional division on the potential application of proportional rules

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Abstract

The most commonly used principle of division of assets and debts is the principle of proportionality, which derived from Aristotle. The main area of application of that rule is the electoral law. An example of another solution can be found in the Lisbon Treaty, which sets out the rule of digressively proportional representation of EU Member States in the European Parliament. The problem is that there are many acceptable solutions. In this paper an attempt is made to determine the boundary conditions in such a way that the rules used for the proportional divisions would also lead to an unambiguous solution in the case of digressively proportional division.

Keywords: Degressive proportionality, European Parliament, fair division, boundary conditions;

1. Introduction

The most common applications are the method of distribution of wealth is a proportional allocation. If it goes to divisible goods, the method is clear, in the case of indivisible goods all of its limitations and pitfalls resulting from the necessity to implement rounding are well known. The current and past electoral laws effectively address the problems that may be encountered in practice using the proportional division. Overview of issues related to the proportional division, for example, the relationship between the electoral legislations is included in the work by Young (1994).

In most cases ratios of representations are determined in proportion to the population of constituencies included in the given body – a region, country, group of countries, etc. When there appear large disparities in constituencies' population and inability to artificially correct these differences, the principle of proportional allocation encounters serious obstacles. This is the case for the European Union and its citizens represented by the European Parliament. Since 2007, the community consists of 27 countries. The least populated country is Malta, whose population in 2012 was 416 thousand, the most populated country with 81,843 thousand citizens is Germany. Such large disparities in population prevent the application of proportional rules along with ensuring fair representation of each country. If one would like to ensure at least one representative for Malta, according to one of the classic rules of proportional

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allocation, the Germans would have to get no less than 197 seats in the parliament consisting of at least 1,208 members.

2. Degressive proportionality

Since the beginning of the European Parliament allocation of seats among the Member States of the European Union (and its predecessors) was not a proportional division. Table 1 shows the number of seats each member state was entitled to in 1979. The fourth column of the table shows that the larger the State (in terms of population), the bigger the ratio of population/number of seats corresponds to it. This ratio indicates how much of the population falls to one representative. For Germany the ratio is 757 058 and is more than 12 times higher than that of 60 377 for Luxembourg.

Table 1. Number of MEPs in 1979

A	B	C	D	E	F	G
Member States	**Population**	**Seats**	**pop/seats (I_n)**	**I_n/ I_{n-1}**	**I_n/ I_1**	**I_1/ I_n**
Germany	61 321 663	81	I_1=757 058		1,0000	1,0000
Italy	56 247 017	81	I_2=694 408	I_2/ I_1=0,9172	I_2/ I_1=0,9172	I_1/ I_2=1,0902
United Kingdom	56 209 039	81	I_3=693 939	I_3/ I_2=0,9993	I_3/ I_1=0,9166	I_1/ I_3=1,0910
France	53 481 073	81	I_4=660 260	I_4/ I_3=0,9515	I_4/ I_1=0,8721	I_1/ I_4=1,1466
Netherlands	13 985 526	25	I_5=559 421	I_5/ I_4=0,8473	I_5/ I_1=0,7389	I_1/ I_5=1,3533
Belgium	9 841 654	24	I_6=410 069	I_6/ I_5=0,7330	I_6/ I_1=0,5417	I_1/ I_{66}=1,8462
Denmark	5 111 537	16	I_7=319 471	I_7/ I_6=0,7791	I_7/ I_1=0,4220	I_1/ I_7=2,3697
Ireland	3 354 700	15	I_8=223 647	I_8/ I_7=0,7001	I_8/ I_1=0,2954	I_1/ I_8=3,3851
Luxembourg	362 261	6	I_9=60 377	I_9/ I_8=0,2700	I_9/ I_1=0,0798	I_1/ I_9=12,5388
TOTAL	259 914 470	410	633 938			

Source: own work based on the European Parliament data.

Columns E, F and G of Table 1 are the measures of proportionality of the division. The division is the more proportional, the closer to unity values in these columns are. Values lower than one in column E signify the fact that the state has more seats than it would have when a proportional distribution was in place (in relation to the state preceding it in the table). The values of column G shows how many times more seats a country has in comparison to the number of seat it would have with the proportional distribution in regards to the largest of the states. Because of the states in Table 1 are arranged from largest to smallest so the last value in column G is in some senses a measure of the total derogation of the given allocation from the proportional distribution. For example, the value of column G for Luxembourg is 12.5388, which means that if the division were to be proportionate, the Germans would have to have 12.5 times more seats than there are in the allocation of Luxembourg.

It is worth noting that the values in column D decrease with the population of the state. This is confirmed by the values of the coefficients in column E (all are less than 1), so during the distribution seats of to the European Parliament for the term from 1979 to 1984 the following principle was used:

If one cannot allocate proportionally, let the small have more.

The European Parliament has operated under its present name since 1962. Since the beginning of the Parliament, for the same reasons already mentioned, it could not be elected using proportional division. Seats distribution was done on the basis of political negotiations before every election, for the period of five years. The allocation of seats was not a result of any strict criterion it was based more on an intuitive sense of justice.

Currently, the European Union has 27 members (in the near future, after ratification of the relevant treaty among the Member States, Croatia will be the next member of the structures). The basic document underlying the legal

framework for selecting the composition of the EP is the Treaty of Lisbon. In the article [9 A] of the Treaty devoted to European Union (with the changes introduced within the Treaty of Lisbon) we read:

2. The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be depressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats. The European Council shall adopt by unanimity, on the initiative of the European Parliament and with its consent, a decision establishing the composition of the European Parliament, respecting the principles referred to in the first subparagraph.

In its session of 11 October 2007, the Parliament adopted a resolution (prepared by the AFCO Committee in its report of October 3, 2007), annex of which states inter alia that:

The principle of depressive proportionality provided for in Article [9a] of the Treaty on European Union shall be applied as follows:

– the minimum and maximum numbers set by the Treaty must be fully utilized to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States;

– the larger the population of a country, the greater its entitlement to a large number of seats;

– the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament.

What is currently meant by digressive proportionality rule is composed of five conditions:

W1. no smaller State shall receive more seats than a larger State,
W2. the ratio population / seats shall increase as population increases,

B1. the minimum number of seats a state receives is set – m,
B2. the maximum number of seats a state receives is set – M,
B3. the total number of seats in Parliament is set – H.

Although there is no provision requires that Parliament had exactly as many MPs as is the upper limit of the Treaty (Lisbon Treaty says only that this number cannot be exceeded), the majority of studies about this issue takes just such an interpretation.

Terms of W1 and W2 are independent of the quantitative situation characterizing the composition of the European Union (the number of countries and their populations). They are more of general characteristics of digressive proportionality. Conditions B1, B2, B3 will be called **boundary conditions**. In the current state of the law it is $m = 6$, $M = 96$, $H = 751$.

Imposition of specific values for the boundary conditions in legal regulations may result in the need to change them in the context of future expansion or demographic changes.

A significant change in the interpretation of the rules of depressive proportionality is presented in the European Parliament's resolution of 13 March 2013 (Gualtieri, Trzaskowski (2013)). It takes into account the proposal contained in the Cambridge Compromise (Grimmett et al (2011)). Article 1 of the resolution states that with the application of the principle of depressive proportionality:

The ratio between the population and the number of seats of each Member State ***before rounding*** *to whole numbers shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member*

State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats.

Discussion of the findings presented in the Cambridge Compromise can be found, inter alia, in articles Dniestrzański (2011) and Grimmett (2011).

3. New proposal - controlling boundary conditions

Let, $p_1 \le p_2 \le ... \le p_n$ be the populations of the succeeding countries, and m_1, m_2,..., m_n the number of seats available to those countries.

Determining minimum and maximum number of seats that the respectively smallest and largest state can (must) receive initiates generation of the suitable depression of division. The W2 condition results in the fact that for each $1 < i < n$ the ratio $\frac{p_i}{m_i}$ must belong to the range $\left(\frac{p_1}{m}, \frac{p_n}{M}\right)$, where p_1 and p_n are the smallest and the largest state's populations respectively.

Allocation of seats is made on the basis of the value of linear function of allocation (this function was considered in the work of Słomczyński and Życzkowski (2012)) $A:[p,P] \to [m,M]$ passing through the points with coordinates (p_1,m) and (p_n,M). It is easy to see that sequence $A(p_1)$, $A(p_2)$, ..., $A(p_n)$ is depressively proportional (with additional natural assumption that $\frac{p}{m} < \frac{P}{M}$) with respect to the sequence of population p_1, p_2, ..., p_n (i.e. it satisfies the conditions W1 and W2). Number of seats can be assigned to a state with a population of p_i, is the rounded value of $A(p_i)$ to an integer value. After rounding the conditions W1 and W2 may not be satisfied, however, taking into consideration the mentioned Article 1 of the European Parliament's resolution is enough for the condition W2 to be met before rounding. Typically, it is:

$$A(p_1) + A(p_2) + ... + A(p_n) = H \neq 751$$

and

$$[A(p_1)] + [A(p_2)] + ... + [A(p_n)] = H \neq 751$$

where $[A(p_i)]$ is the rounded number $A(p_i)$ to an integer (down, up, or the nearest).

Let it be for the set m and M that:

$$H_{\min} = \lfloor A(p_1) \rfloor + \lfloor A(p_2) \rfloor + ... + \lfloor A(p_n) \rfloor,$$
$$H_{\max} = \lceil A(p_1) \rceil + \lceil A(p_2) \rceil + ... + \lceil A(p_n) \rceil,$$
$$H_a = [A(p_1)]_a + [A(p_2)]_a + ... + [A(p_n)]_a,$$

where, $\lfloor x \rfloor$, ($\lceil x \rceil$) represent the rounding of the number x down (up) to the nearest integer and $[x]_a$ is the rounding of the number x to the nearest integer.

Values $H_{\min}$ and $H_{\max}$ will be called **the lower and upper limit** for the function A .

Definition

1 We say that the system of boundary conditions B1 - B3 is **admissible** if $H \in [H_{\min}, H_{\max}]$. Otherwise, we say that the system is **inadmissible**.

2 We say that the system of boundary conditions B1 - B3 is **consistent** if $H = H_{\min}$ or $H = H_a$ or $H = H_{\max}$.

The above definition regards as admissible the boundary conditions that allow the use of allocation function A for the distribution of the seats. It verifies in a sense the validity of the imposed constraints (boundary conditions) in a given system of states and their populations. If $H = H_{\min}$ or $H = H_{\max}$ or $H = H_a$ (consistent system) then when aiming at allocating seats in accordance with the described method, there is no choice - the allocation was made. If, however, it is $H \in [H_{\min}, H_{\max}]$, but the system is not consistent, then there is a distribution of seats in which for each $1 \le i \le n$ the number of seats for a state with population of p_i varies from $A(p_i)$ by less than 1.

Table 2 lists the distribution of seats in the current legislature, distribution of seats adopted by the Parliament for the next term and the allocation of seats to resulting from implementation of the function A with rounding down (column F) and up (column G). The statement includes Croatia, which is likely to become a member of the European Union in the current term. Total of 12 observers in Parliament at the disposal of Croatia was treated here as the number of members of Parliament.

Table 2. Number of MEPs: current, 2014 – 2019; lower and upper limit for $m = 6$ and $M = 96$

A	B	C	D	E	F	G
member state	**population**	**current**	**2014-2019**	$A(p_i)$	$\lfloor A(p_i) \rfloor$	$\lceil A(p_i) \rceil$
Germany	81 843 743	99	96	96,00	96	96
France	65 397 912	74	74	77,82	77	78
United Kingdom	62 989 550	73	73	75,16	75	76
Italy	60 820 764	73	73	72,76	72	73
Spain	46 196 276	54	54	56,60	56	57
Poland	38 538 447	51	51	48,14	48	49
Romania	21 355 849	33	32	29,14	29	30
Netherlands	16 730 348	26	26	24,03	24	25
Greece	11 290 935	22	21	18,02	18	19
Belgium	11 041 266	22	21	17,74	17	18
Portugal	10 541 840	22	21	17,19	17	18
Czech Republic	10 505 445	22	21	17,15	17	18
Hungary	9 957 731	22	21	16,55	16	17
Sweden	9 482 855	20	20	16,02	16	17
Austria	8 443 018	19	18	14,87	14	15
Bulgaria	7 327 224	18	17	13,64	13	14
Denmark	5 580 516	13	13	11,71	11	12
Slovakia	5 404 322	13	13	11,51	11	12
Finland	5 401 267	13	13	11,51	11	12
Ireland	4 582 769	12	11	10,61	10	11
Croatia	4 398 150	12	11	10,40	10	11
Lithuania	3 007 758	12	11	8,86	8	9
Slovenia	2 055 496	8	8	7,81	7	8
Latvia	2 041 763	9	8	7,80	7	8
Estonia	1 339 662	6	6	7,02	7	8
Cyprus	862 011	6	6	6,49	6	7
Luxembourg	524 853	6	6	6,12	6	7
Malta	416 110	6	6	6,00	6	6
TOTAL	508 077 880	766	751	716,69	$H_{\min}$=705	$H_{\max}$=731

Source: own work.

Terms of the allocation of seats in Parliament of 8th term (2014 – 2019) ($m = 6, M = 96, H = 751$) form a inadmissible system - we have here $H_{\min} = 705, H_{\max} = 731$ (columns F and G of Table2). Table 3 shows all the values m and M, which with value $H = 751$ create admissible systems.

Table 3. Lower and upper limits creating an admissible system for $H = 751$

m **and** M		m **and** M		m **and** M	
$m = 0$	$M \in \langle 121,125 \rangle$	$m = 9$	$M \in \langle 90,92 \rangle$	$m = 18$	$M \in \langle 57,60 \rangle$
$m = 1$	$M \in \langle 118,121 \rangle$	$m = 10$	$M \in \langle 85,89 \rangle$	$m = 19$	$M \in \langle 53,57 \rangle$
$m = 2$	$M \in \langle 114,117 \rangle$	$m = 11$	$M \in \langle 82,85 \rangle$	$m = 20$	$M \in \langle 50,53 \rangle$
$m = 3$	$M \in \langle 110,114 \rangle$	$m = 12$	$M \in \langle 78,82 \rangle$	$m = 21$	$M \in \langle 46,50 \rangle$
$m = 4$	$M \in \langle 107,110 \rangle$	$m = 13$	$M \in \langle 75,78 \rangle$	$m = 22$	$M \in \langle 42,46 \rangle$
$m = 5$	$M \in \langle 103,107 \rangle$	$m = 14$	$M \in \langle 71,75 \rangle$	$m = 23$	$M \in \langle 39,42 \rangle$
$m = 6$	$M \in \langle 100,103 \rangle$	$m = 15$	$M \in \langle 67,71 \rangle$	$m = 24$	$M \in \langle 35,39 \rangle$
$m = 7$	$M \in \langle 96,100 \rangle$	$m = 16$	$M \in \langle 64,67 \rangle$	$m = 25$	$M \in \langle 31,35 \rangle$
$m = 8$	$M \in \langle 92,96 \rangle$	$m = 17$	$M \in \langle 60,64 \rangle$	$m = 26$	$M \in \langle 27,31 \rangle$

Source: own work.

For the currently adopted size of the European Parliament ($H = 751$), there are 123 admissible systems. Among the admissible systems there are 16 consistent systems. All systems are consistent (for $H = 751$) are provided in Table 4.

Table 4. Consistent systems for $H = 751$

1	$m = 0$	$M = 121$	$H_{\max} = 751$	9	$m = 17$	$M = 62$	$H_a = 751$
2	$m = 2$	$M = 116$	$H_a = 751$	10	$m = 19$	$M = 57$	$H_{\min} = 751$
3	$m = 5$	$M = 103$	$H_{\min} = 751$	11	$m = 19$	$M = 55$	$H_a = 751$
4	$m = 8$	$M = 92$	$H_{\max} = 751$	12	$m = 20$	$M = 53$	$H_{\min} = 751$
5	$m = 9$	$M = 91$	$H_a = 751$	13	$m = 20$	$M = 51$	$H_a = 751$
6	$m = 10$	$M = 85$	$H_{\max} = 751$	14	$m = 21$	$M = 50$	$H_{\min} = 751$
7	$m = 13$	$M = 77$	$H_a = 751$	15	$m = 22$	$M = 46$	$H_{\min} = 751$
8	$m = 14$	$M = 75$	$H_{\min} = 751$	16	$m = 25$	$M = 31$	$H_{\max} = 751$

Source: own work.

The presented consistent systems with 751-member-Parliament form a wide range of possibilities of using the proposed method. The consistent division for $m = 0$ and $M = 121$ is close to the proportional division. The results of the simulation say that with this division six of the largest EU Member States would receive 527 seats (about 70.17%). At the same time the population of these countries accounts for 70.03% of the population of all countries of the community. The other extreme consistent division ($m = 25, M = 31$) is very close to the equal division, as many as 19 countries would receive in this case 26 seats each.

Among all consistent systems the system $m = 8$ and $M = 92$ is closest to provisions of the Treaty of Lisbon ($m \geq 6$ and $M \leq 96$). Table 5 compares the distribution of seats of the 8th term with a division for $m = 8$ and $M = 92$ among which there is a consistent system with the function A values rounded up.

Table 5. Number of MEPs: 2014 – 2019; lower and upper limit for $m = 8$ and $M = 92$

A	B	C	D	E	F	G
member state			$m = 8$ **i** $M = 92$			
	2014-2019	$A(p_i)$	$\lceil A(p_i) \rceil$	$\lfloor A(p_i) \rfloor$	$\lceil A(p_i) \rfloor_a$	**difference D-B**
Germany	96	92,00	92	92	92	-4
France	74	75,03	76	75	75	2
United Kingdom	73	72,55	73	72	73	0
Italy	73	70,31	71	70	70	-2
Spain	54	55,23	56	55	55	2
Poland	51	47,33	48	47	47	-3
Romania	32	29,60	30	29	30	-2
Netherlands	26	24,83	25	24	25	-1
Greece	21	19,22	20	19	19	-1
Belgium	21	18,96	19	18	19	-2
Portugal	21	18,45	19	18	18	-2
Czech Republic	21	18,41	19	18	18	-2
Hungary	21	17,84	18	17	18	-3
Sweden	20	17,35	18	17	17	-2
Austria	18	16,28	17	16	16	-1
Bulgaria	17	15,13	16	15	15	-1
Denmark	13	13,33	14	13	13	1
Slovakia	13	13,15	14	13	13	1
Finland	13	13,14	14	13	13	1
Ireland	11	12,30	13	12	12	2
Croatia	11	12,11	13	12	12	2
Lithuania	11	10,67	11	10	11	0
Slovenia	8	9,69	10	9	10	2
Latvia	8	9,68	10	9	10	2
Estonia	6	8,95	9	8	9	3
Cyprus	6	8,46	9	8	8	3
Luxembourg	6	8,11	9	8	8	3
Malta	6	8,00	8	8	8	2
TOTAL	751	736,11	$H_{\max} = 751$	$H_{\min} = 725$	$H_a = 734$	

Source: own work.

4. Summary

Terms of W1 and W2 determining the digressive proportionality are very general. They allow many possible solutions. Even taking into consideration the Treaty's additional boundary conditions on the minimum, maximum and total number of deputies one can show that, regarding the population data of 2006, the number of such divisions exceeds 189 million (Łyko et al (2012)). An important feature of the construction of specific divisions is therefore boundary conditions B1 - B3. They clarify the right depression. It may also happen, that they are somewhat contradictory. If one accepts the supposition that the sought digressively proportional division should not, with accuracy to the nearest rounding, be very different from the natural allocation obtained with the defined function A it may occur that there is no such division. Referred to in article, the admissibility of boundary conditions is a formal record helping to avoid the said conflict.

Calculations performed on the rules formulated for the distribution of seats among the Member States of the European Parliament attest to their internal contradictions. Equalities, $m = 6$, $M = 96$ and $H = 751$ do not constitute an acceptable or even consistent system of boundary conditions. Therefore, all possible pairs of values were given for m and M, which for the fixed $H = 751$ which constitute, for population data p_1, p_2, …, p_n from the 2012 such an admissible system. They provide 123 admissible systems and 16 consistent systems. Among the found ones, the couple $m = 8$, $M = 92$ differs the least from the restrictions indicated by the Lisbon Treaty ($m \geq 6$, $M \leq 96$).

An allocation was found of 751 seats, which for the couple $m = 8$, $M = 92$, is obtained by rounding the value $A(p_i)$ up to the nearest integer. It may be noted that the proposed solution is relatively close to the division of seats for the term 2014-2019 adopted by the European Parliament (which is still waiting for approval by the European Council). Apart from Germany (where the difference of four seats is a direct consequence of the restriction $M = 92$) the difference in the number of seats attributed to individual members of the Union shall be not more than 3.

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